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Pricing take-or-pay provisions in gas markets with limited liquidity

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ABSTRACT

Long-term contracting is the traditional governance mechanism to deal with the transaction costs associated with the specificity of gas industry assets. Long-term contracts have been often used to allocate risks among players, and to that end, they often include take-or-pay provisions. These clauses specify that buyers take the volume risk, as they are obliged to pay for a minimum amount of gas consumption. In exchange, buyers pay a predefined price, supposedly lower than the risk-neutral expectation of short-term gas prices. In that view, if the buyer is able to resell that gas in the short term, the contract is an effective hedge against short-term volatility. Otherwise, the contract does not act as a hedge but it becomes a sunk cost. The corresponding power producers' behavior involves not only output decisions but also financial decisions. To analyze that situation, this paper develops a new quantitative methodology that allows comparing risk-neutral valuations of gas and power markets decisions. We test the model in a real-size system, and show the additional cost of the power system associated with a possible illiquidity of the short-term gas market.

Key Words: Long-term contracts; Gas-power interaction; Take-or-pay provisions; Risk-neutral pricing

1 Introduction

The usual way of purchasing natural gas for power production is the long-term contract. This is related to the specificity of natural gas consumption, see for instance (Masten & Crocker, 1985) or (Mulherin, 1986) for a transaction cost perspective. In that context, the importance of short-term flexibility was pointed out in (Creti & Villeneuve, 2004), and this aspect of the problem takes center stage when analyzing the interaction between gas and power industries. The time scope for decision-making in power industries is often shorter than the one for gas industries, so the ability to pursue wait-and-see strategies is more valuable than when the gas industry is considered alone.

On the other hand, power producers often enter into long-term contracts to purchase the fuel required to fire power plants, and these contracts usually contain take-or-pay provisions, which penalize the buyer for not purchasing a minimum amount of fuel over a determined period. In particular, power producers usually purchase their natural gas supplies through this kind of contract. Typically, take-or-pay contracts specify a minimum amount of fuel that must be purchased in the contract period, and the corresponding unit price for fuel purchases. Although these contracts clauses actually vary from one contract to another, the essential characteristic of take-or-pay contracts is the risk transfer from the seller to the buyer, in particular the risk of a decline in production of gas-fired units, in exchange of a production commitment on the seller's side.

Gas and power systems, on the other hand, are technically difficult to operate, and both of them require markedly tight short-term coordination (power systems require tighter coordination). In the US gas systems, as long-term gas contracts are typically associated face frequent imbalances, shippers need considerably complex combinations of gas trade arrangements and the associated transmission rights, (Costello, 2006). In EU, it is usual that some degree of flexibility in gas balances is given to shippers for free, as discussed by (Lapuerta & Moselle, 2002) and (Hallack & Vazquez, 2013). In power systems, market designs need to rely on ancillary services, see for instance (Joskow & Schmalensee, 1988) or (Stoft, 2002), which can be interpreted as system flexibility. The discussion on the amount of necessary locational prices can be put under the flexibility header as well, see for instance (Hogan, 2002).

Moreover, renewable energy will need increasing amounts of backup generation, which in turn creates the need for flexibility in the gas system. The interaction between the two industries will have a prominent role in the efficiency achieved in the renewable energy process. Market design is not neutral to those problems. Both gas and power markets are characterized by complex technical characteristics that motivate special needs of regulation. Such market design needs to take into account cross-commodity trading, in order to avoid being a barrier to trade. One relevant effect, identified in (Vazquez &

Hallack, 2014), is that free short-term flexibility for gas consumption represents cross-subsidies to flexible gas consumers, namely CCGTs.

In this paper, we are concerned with a situation where the holder of a long-term gas contract is a power producer, and this power producer needs to deal with imbalances in its long-term position in the short run. Besides, free flexibility in the gas system is not enough to cope with the imbalances of long-term gas purchases. Put differently, we are concerned with the problem that a power producer needs to commit in the very long run to a certain volume of gas consumption, but decisions on power production are typically taken in the short run.

In the previous context, two situations may be found. The first one is characterized by the fact that power producers are able to renegotiate gas purchases in the short run. This situation assumes a liquid short-term gas market where imbalances may be reallocated. Hence, the response to an imbalance between gas and power decisions is absorbed by the most efficient market, being that either power or gas market.

In the second situation, short-term gas markets are illiquid, and so the gas cannot be reallocated. Hence, any imbalance between power and gas profiles is absorbed by the power industry, even in the case where the most efficient way to cope with the imbalance is the response of gas consumption. Consequently, power producers consider a risk premium associated with the illiquidity of short-term market transactions. This risk premium is related to the difficulties in the adaptation of long-term gas contracts to short-term power contracts. In this context, the power producer will ask for a reduced gas price that compensates for the additional risk of the contract.

In that view, the typical problem faced by the power producer is to determine the adequate contract price given the minimum amount of gas of the take-or-pay provision. For the sake of simplicity, it is possible to think of the problem as the trade-off between a cheap price with a production obligation, and a flexible operation of power plants facing the natural gas spot price.

The strategy we follow in this paper is to describe in detail the fine structure of the risks borne by power generation portfolios. This problem can be modeled in the context of the fundamental-structure model presented in (Vazquez & Barquin, 2013). In fact, adding a constraint representing the obligation of consuming the gas specified in the contract will allow obtaining the cost of this constraint, by means of the corresponding shadow price. Thus, the model obtains a set of scenarios for the shadow price representing the cost of inflexibility, which can be compared to the cost of the flexible contract.

The program for this paper is the following. After this introduction, in section 1, we propose a methodology to price power contracts designed to represent both complex cost structures and strategic behavior. In section 2, we use the previously defined methodology to model the cost of the inflexibility associated with long-term gas contracts when producing electricity. Section 2 describes the complete pricing model, and section 3 presents an application to a real-size power system. Section 4 concludes.

1. The modeling strategy

Absence of arbitrage can be thought of as the fundamental tool to describe financial markets. The no-arbitrage price of any financial contract represents players' valuation of the uncertain future income stream that will result from the contract. As a part of the calculation of these valuations, a risk-neutral probability might be defined. It modifies the real probability to incorporate the effects of players' preferences (risk aversion, etc.), see for instance (Duffie, 2010). Hence, the problem of pricing financial products can be tackled by obtaining the uncertain income associated to the financial contract and calculating their expected value under the risk-neutral probability.

One of the ways of calculating these risk-neutral probabilities is based on the idea that the prices observed in the financial market represent its equilibrium and, thus, can be used to estimate from them a risk-neutral probability that represents the aggregated perceptions of market players. In practice, this is often done by selecting a certain class of stochastic processes, which is characterized by as many parameters as it is required to represent the shape of power price distributions. These parameters are then calibrated to match actual prices quoted in the market.

This paper studies the pricing problem in energy markets. In particular, we will analyze power price dynamics, and in this case, several features need to be taken into account. First, the complex structure of production costs and the impossibility of economic storage make the power price distribution considerably difficult to represent. In order to aid price representation, financial econometrics literature has proposed the use of auxiliary variables to describe electricity prices. The logic for the approach, which can be traced back to (Box & Cox, 1964), is based on the assumption that a complex behavior can be described by the transformation of several random processes, each of them described by some simpler dynamics. Moreover, the representation of the information used to price contracts is difficult in power markets. As it is a non-storable commodity, the usual assumption that spot prices generate a filtration containing all the available information is dubious, as pointed out in (Benth & Meyer-Brandis, 2009). Forward-looking information is central in pricing electricity forwards, but it is not contained in any filtration generated by spot prices.

When applied to power markets, the central idea behind this approach is to describe, instead of power prices, the evolution of fundamental drivers by means of the definition of the function transforming fundamental drivers into electricity prices. This is the modeling strategy pursued by (Eydeland & Geman, 1999), where the power price is defined as a function of a deterministic supply function and the system demand. Along the same lines, (Skantze, Gubina, & Ilic, 2000) model supply and demand using Principal Component Analysis. (Barlow, 2002) proposes the use of supply and demand as fundamental variables and defines the transformation function using a Box-Cox transformation, whereas (Burger, Klar, M ller, & Schindlmayr, 2004) consider a nonparametric transformation. Finally, the definition of the transformation function using fundamentals allows introducing forward-looking information, as future reserve margins, see (Mount, Ning, & Cai, 2006) and (Anderson & Davison, 2008).

Nonetheless, the transformation of power price fundamentals is extremely difficult to define. The characteristics of electricity production result in complex transformation functions, so that their statistical definition depends again on a large number of parameters, which requires a large amount of historical data. An alternative approach consists in taking advantage of the knowledge of the market structure to simplify the estimation of the transformation function. This is the idea behind the methodology introduced in (Eydeland & Wolyniec, 2003). (Fleten & Lemming, 2003) uses a unit-commitment model to fit forward curves using bids and asks. (Tipping, Read, & McNickle, 2004) uses a unit-commitment model to represent the influence of water resources in New Zealand spot prices. In the financial econometrics literature, several authors have analyzed the problem of considering additional explanatory variables. (Cartea & Villaplana, 2008) introduced an additional process representing the available capacity to adjust the bid curve. (Aid, Campi, Huu, & Touzi, 2009) consider the risk neutral process of the cost-based transformation. (Howison & Coulon, 2009) use both fuel prices and capacity processes. (Aid, Campi, & Langrené, 2013) analyzes analytically the pricing problem using, besides a cost-based transformation, a scarcity function.

Furthermore, the transformation is further complicated by the frequent existence of horizontal concentration. From this standpoint, the transformation function needs to represent not only complex cost structures but also the possible exercise of market power (including forward-looking information about strategies). Game theory models, and especially the ones based on static games, have been extensively used to represent power spot markets. However, their results depend on model assumptions and hence they alone may not describe the price dynamics in a robust quantitative manner.

We will use thus a methodology able to capture the complexities of power markets and incorporate them into the framework of risk-neutral pricing. To that end, we split up the power price model into two different components, along the lines of (Schwartz & Smith, 2000). On the one hand, the component aimed at representing costs and market power, which will use a static, non-cooperative game to model the transformation function. On the other, the component representing short-term deviations from the first component. Hence, the representation of forward-looking information about both costs and strategies will be included in the first component. To that end, we use the methodology developed in (Vazquez & Barquin, 2013).

1.1 The risk-neutral transformation function

Our modeling strategy may be motivated from the stochastic discount factor framework, proposed in (Hansen & Richard, 1987) for two-period models. To that end, let us denote the payoff of any financial contract written on electricity and expiring at $t+1$ by $v_{t+1}^{contract}(p_{t+1}^{power})$, where p_{t+1}^{power} is the power spot price. Thus, its price at time t, $p_t^{contract}(v_t^{contract})$, can be calculated as:

$$p_t^{contract}(v_t^{contract}) = E_t\left[\bar{\varepsilon}_{t+1}^{power} v_{t+1}^{contract}\right]$$

where $\bar{\varepsilon}_{t+1}^{power}$ is the stochastic discount factor used to price the contracts at time t, and $E_t[\cdot]$ denotes the conditional expectation at time t. To consider more than one future period, it is possible to rely on the extension of the stochastic discount factor methodology to the multi-period setting developed in (Garcia, Ghysels, & Renault, 2010). Hence, we will denote the corresponding discount factor by:

$$\varepsilon_{t+i}^{power} = \left(\bar{\varepsilon}_{t+1}^{power}\right)\left(\bar{\varepsilon}_{t+2}^{power}\right)\cdots\left(\bar{\varepsilon}_{t+i}^{power}\right)$$

where i is a natural number representing a certain future period, and $\bar{\varepsilon}_{t+1}^{power}$ is the discount factor defined for the two-period setting. Let us also consider the price of a forward contracts written on power $p_{t,t+T}^{F}$, where T denotes the time to the expiration of the contract:

$$p_{t,t+T}^{F} = E_t\left[\varepsilon_{t+T}^{power} p_{t+T}^{power}\right]$$

In this context, we analyze the transformation $F(\cdot)$, which gives power prices from fundamental drivers. In addition, we explicitly take into account that the transformation function may not contain all the relevant elements of the price dynamics. So the previous expression can be rewritten as

$$p_{t,t+T}^{F} = E_t\left[\varepsilon_{t+T}^{power}\left\{F\left(\varepsilon_{t+T}^{j} x_{t+T}^{j}\right) + y_{t+T}^{power}\right\}\right]$$

where x_{t+T}^{j} represents each of the underlying factors (fuel prices, demand...), ε_{t+T}^{j} the corresponding discount factor, and y_{t+T}^{power} a certain stochastic process representing deviations from the behavior described by the fundamental transformation[1]. On the other hand, the transformation function $F(\cdot)$ will be defined in this paper by means of a model of the spot market behavior. It is aimed at representing in detail both the production costs and the strategic interaction among spot market players. To do so, such

[1] Note that the above expression assumes that no state variables are considered in the model for fundamental drivers.

model will be calibrated to represent risk-neutral power prices, so the transformation function may be interpreted as a static transformation from risk-neutral fundamental drivers to risk-neutral power prices.

Put differently, the discount factor $\varepsilon_{t+T}^{power}$ can be interpreted as representing electricity-specific risks, so the pricing model can be expressed in the following way:

$$p_{t,t+T}^{F} = E_t\left[\bar{F}\left(\varepsilon_{t+T}^{j} x_{t+T}^{j}\right) + \varepsilon_{t+T}^{power}\, y_{t+T}^{power}\right]$$

where $\bar{F}(\cdot)$ is the risk-neutral transformation function $\varepsilon_{t+T}^{power} F\left(\varepsilon_{t+T}^{j} x_{t+T}^{j}\right)$. In that view, (Vazquez & Barquin, 2013) showed that this methodology allows extending the relationship between fundamentals and spot prices to the representation of electricity forward prices.

1.2 Strategic interaction as a model for the transformation function

In the rest of the paper, we first describe the model used by the pricing methodology to describe power prices. In particular, in this subsection, we describe the transformation function. After this, the next subsection discusses the distributional effects of market power, in order to justify the fundamental-structure approach. The next section will develop, from the model introduced in this section, the representation of take-or-pay clauses.

We will consider that the spot market equilibrium is defined by the solution of a static, non-cooperative game. Formally, the game is defined by the interaction of firms, each of whom solves a profit-maximizing problem taking into account that their decisions can effectively modify the market price. In addition, the market operator clears the market and calculates the price. For the sake of simplicity, we will consider the aggregate output of each firm. Let us define:

- q_i is the total output of firm i
- $C_i(q_i)$ is the generation cost of firm i
- q_i^{max} is the maximum output of firm i
- ρ_i^{min} and ρ_i^{max} are the Lagrange multiplier corresponding to minimum and maximum output constraints, respectively
- π is the equilibrium price

Thus, each firm solves the following problem:

$$\begin{aligned} &max \quad \pi(q_i)q_i - C_i(q_i) \\ &s.t. \quad 0 \le q_i \le q_i^{max} \qquad : \rho_i^{min}, \rho_i^{max} \end{aligned}$$

Besides, in order to solve the Nash game we need equations that explain the behavior of the market operator. In this case, we will consider that the operator's clearing process is represented just by imposing that demand is equal to supply. This implies that we are considering an inelastic demand. Formally, $\sum_i q_i = D$. The set of equations that describe the Nash equilibrium are

- Each firm's optimality with respect to output decisions (one optimality per firm)

$$\pi(q_i) + \frac{\partial \pi}{\partial q_i} q_i - \frac{\partial C_i(q_i)}{\partial q_i} - \rho_i^{max} + \rho_i^{min}$$

- Each firm's maximum output constraint

$$0 \leq q_i \leq q_i^{max}$$

- Each firm's complementarity conditions ($A \perp B$ denotes that A and B are complementary)

$$\left(q_i - q_i^{max}\right) \perp \rho_i^{max}$$

$$\left(0 - q_i\right) \perp \rho_i^{min}$$

The equilibrium point, thus, has to fulfill the set of equations defined by the optimality conditions of every market participant, plus the market clearing equation $\sum_i q_i = D$. In order to solve the problem, we assume that the cost curve is known, so that $\frac{\partial C_i(q_i)}{\partial q_i}$ is known as well. We also assume that $\frac{\partial \pi}{\partial q_i}$ is a known parameter of the problem.

One of the main elements of this class of oligopoly model is to define the previous term representing the ability of firms to manipulate prices. A first alternative is the Cournot model: market players choose their quantities in order to maximize their profits, and considering that competitors do not react to output decisions. Therefore, price changes associated with output decisions of a certain agent are related only to changes in the quantity demanded. Thus, the previous derivative is defined by the elasticity of the demand, so the model cannot deal with inelastic demands. (Borenstein & Bushnell, 1999) is an example of the application of Cournot competition to describe strategic interaction in power systems.

A refinement of the Cournot model is the supply function equilibrium. Originally, (Klemperer & Meyer, 1989) developed the concept of supply function equilibrium as a compromise between price and quantity competition. They suggest that in an uncertain environment firms would not want to commit with either of these strategies. Instead, firms would specify supply functions, ie functions specifying the bid price corresponding to each possible output. Compared to the Cournot model, the supply-function equilibrium implies that the ability to manipulate the price is no longer demand's slope. Market players take into account rivals' reactions, so that the price sensitivity is the residual demand's slope (allowing the analysis of inelastic demands). The additional difficulty of the model is that the residual demand's slope is part of the equilibrium definition. Supply-function equilibrium was first adopted to study power markets in (Green & Newbery, 1992) and (Bolle, 1992).

Supply-function equilibrium, although providing many important insights, is often difficult to solve. This is the motivation for conjectured-supply-function equilibrium. The central idea behind this approach is to define a parameterized supply function for each producer, so that the number of available decisions is reduced. A typical example is to use linear functions with known slope. That is, the ability of firms to affect spot prices is a constant and known parameter, see (Day, Hobbs, & Pang, 2002) for the application of this approach to electricity systems[2]. Hence, we will define $\theta_i = -\frac{\partial \pi}{\partial q_i}$

From the viewpoint of our pricing methodology, the approach based on conjectured supply functions is especially appropriate. The pricing methodology is aimed at capturing oligopolistic behavior through the calibration of the model. With this approach, we only need to calibrate one parameter. The fact that this model fails to represent multi-period effects is not an important limitation of the model, as the strategic component is supposed to be in our methodology a static one.

Our approach to solve this equilibrium problem builds on the analysis developed in (Hashimoto, 1985). The central idea behind that work is that it is possible to use a single optimization program as a representation of the strategic interaction, because the optimality conditions of the appropriate optimization problem are the same as the equilibrium conditions of the previous game. The main advantage is that the optimization problem is easier to solve. (Barquin, Centeno, & Reneses, 2004) extended the methodology to consider conjectured supply functions.

It is easy to check that the equilibrium conditions defined above are the same as the first-order optimality conditions of the following quadratic program:

[2] Note that this model is the same as the conjectural variations approach, which can be traced back to (Bowley, 1924). We motivate the approach from the supply function model to highlight the static nature of the game considered.

$$\begin{aligned} min \quad & \sum_i \theta_i \ q_{i,t}^2 + C \ \ (q_i \ ) \\ s.t. \quad & 0 \le q_i \ \le q_i^{max} \qquad \qquad : \rho_i^{min}, \rho_i^{max} \\ & \sum_i q_i \ = D \qquad \qquad \quad : \pi \end{aligned}$$

1.3 Distributional effects of strategic interaction

We showed above that representing the market-clearing price as a certain calibration of the marginal plant's cost implies simplifying the game between producers (it does not consider market power). This is often not approximate enough. It is worth to analyze the results implied by the equilibrium model in some detail, since they play a key role in the characteristics of the price process proposed in this paper. In particular, from the analysis of the equilibrium conditions, it is possible to identify the expression $-\frac{\partial c_i \ (q_i \ )}{\partial q_i} - \rho_i^{max} + \rho_i^{min}$ as the marginal cost of the firm i. Assuming that the firm is producing, $\rho_i^{min} = 0$, if the output of the firm is below its limits then the maximum output constraint is not active and its Lagrange multiplier is equal to zero, $\rho_i^{max} = 0$. This implies that the firm's production is at the margin. If the maximum output constraint is binding, the Lagrange multiplier is greater than zero, $\rho_i^{max} > 0$, and the firm is below the margin. Consider first that there is no opportunity to manipulate the price, or equivalently, the market is perfectly competitive. Then, $\frac{\partial \pi}{\partial q_i} = 0$ and the equation becomes $\pi(q_i \ ) = \frac{\partial c_i \ (q_i \ )}{\partial q_i} + \rho_i^{max} - \rho_i^{min}$, the traditional "price is equal to marginal cost" result. The term $\frac{\partial \pi}{\partial q_i} q_i$ shows the incentives for price manipulation that arises in the market. This value makes the price higher than the marginal cost (note that $\frac{\partial \pi}{\partial q_i}$ is negative). $\frac{\partial \pi}{\partial q_i}$can be interpreted as the ability of the firm to modify the prices, while $q_i \ \ g_i$ measures how much the firm benefits from that increment.

The strategic term implies that the bid price of the marginal plant depends not only on its own cost, but also on the production of the rest of the generation portfolio. Consequently, the merit order is not known*a priori* but it is determined through the solution of the game. To see the effects of strategic behavior on price distributions, let us assume competitive behavior, so that the price is obtained using the aggregate supply curve:

;

Besides, if we consider that the conjectured variations $\frac{\partial\pi}{\partial q_i}$ of all firms are the same, $\frac{\partial\pi}{\partial q_i} = \theta$. The residual between an oligopoly model and a model based only on production costs can be expressed by

$$\text{Strategic Term} = \theta \sum_i q_i \ = \theta D$$

which is a linear function of the demand. Assuming that the system demand is approximately normal, this term would be an increasing function of a normal distribution, and thus it would be an additional source of fat tails.

2 Pricing take-or-pay provisions

In the context of fundamental structure models, take-or pay obligations can be represented by a constraint on the gas consumption of gas-fired plants. To do that, we extend slightly our equilibrium description from the previous section, and consider a sequence of spot markets where the previously described equilibrium happens. All variables represented previously will be dependent on an additional index, t, to represent the point in time when the equilibrium model is considered. Hence, for instance, the production of firm i at time t is represented by $q_{i,t}$. The strategic interaction does not change, as we do not consider inter-temporal effects of strategic behavior. Besides, in order to introduce the model for take-or-pay provisions, we will use $q_{i,t}^{gas}$ to refer to the gas-fired power production of firm i at time t. In addition, let us define $E_i^{contract}$ as the minimum energy that needs to be produced using gas-fired power plants by firm i during the simulation scope –this minimum energy is a representation of the take-or-pay clause. Hence, we can express take-or-pay provisions by the following constraint:

$$\sum_t q_{i,t}^{gas} \geq E_i^{contract}$$

Following the same idea, and adding the parameter t to the definition of the optimization problem, we represent the strategic interaction in the sequence of games in the simulation scope by means of the following program:

$$
\begin{aligned}
\min \quad & \sum_{i,t} \theta_{i,t} q_{i,t}^2 + C_t\ (q_{i,t}) & \\
s.t. \quad & 0 \le q_{i,t} \le q_{i,t}^{max} & : \rho_{i,t}^{min}, \rho_{i,t}^{max} \\
& \sum_i q_{i,t} = D_t & : \pi_t \\
& \sum_t q_{i,t}^{gas} \ge E_i^{contract} & : \rho_i^{gas}
\end{aligned}
$$

The approach suggested in this section is based on considering the Lagrange multiplier ρ_i^{gas} to represent the cost of an inflexible contract. That is, the shadow price is the cost of being obligated to produce an amount of electricity greater than$E_i^{contract}$. Hence, power producers face this extra cost when entering into a take-or pay contract, so that its price must compensate for this cost. However, the particular value of the shadow price depends on the values that fuel prices and system demand take. In other words, power producers face a distribution of flexibility costs, so that they face a different value of the flexibility cost for each scenario of fundamental drivers.

In addition, the alternative to the take-or-pay contract is purchasing their gas supplies in the gas spot market. Therefore, it is possible to calculate the price that makes indifferent to enter into the take-or-pay contract or buying in the spot market. Let p_t^{gas} be the gas spot price at time t, so that the indifference price of the contract is

$$p_t^{contract} = p_t^{gas} + \rho_i^{gas}$$

where it is implied that ρ_i^{gas} takes only negative values. The previous equation considers a time-varying contract price. Usually, take-or-pay contracts specify a unique price for the whole contract period. Therefore, the firm entering into take-or-pay contracts will typically face some scenarios with a contract price higher than required to compensate for the additional constraint, and some scenarios with a lower price than required.

From this viewpoint, the take-or-pay contract will hedge the firm against high spot prices. Hence, as in the case of forward pricing, the final contract price will depend on producers’ risk preferences. This representation implicitly assumes that each agent’s gas consumption is associated with firing her power plants. Nonetheless, if the power producer represented by this constraint only operates in gas markets similar to the spot market, the model captures the joint optimization of gas and electricity portfolio, by means of risk-neutral probabilities. Actually, one of the main advantages of fundamental structure models is that they describe relation between preferences in both markets.

Put differently, the model calibration process captures the relation between risk-neutral –and hence optimal– power prices and the corresponding gas prices. Hence, as long as the gas market where the power producer operates is close enough to the market used to calibrate the fundamental structure model, the model implies the portfolio optimization. A typical example is a firm operating both in a retail gas market and in a wholesale power market. If the company has no access to international gas spot markets, it would imply that the company is constrained to purchase her gas supplies in the domestic gas market.

However, it is usual that the firm has access to international spot markets but the retail gas market is isolated, so that her gas purchases can be done at the international price, and then sold at a different price –for instance, because of transportation costs, market power, liquidity constraints, etc. In such a case, the firm would face the problem of choosing between retail gas price and wholesale power price. From the viewpoint of price optimization, the opportunity cost can be modeled as the international gas price. Thus, the fundamental structure model describes a wide range of decisions regarding the optimization of the joint portfolio.

2. The model to represent electricity spot prices

Following (Vazquez & Barquin, 2013), we will represent the electricity spot market price using two components. The first one, representing dynamics with longer periods, will be given by the price obtained from the oligopoly model described in the previous section, π_t . The second component will represent stochastic perturbations around the dynamics given by the oligopoly model. Thus, we will define the spot market price as p_t^{power}, and it will be made up of two components:

$$p_t^{power} = \pi_t + y_t$$

The general scheme of the model is represented in Figure 1. The first step of the model definition is to describe the evolution of the primary drivers, represented in the left part of Figure 1, which are the basic input for the model.

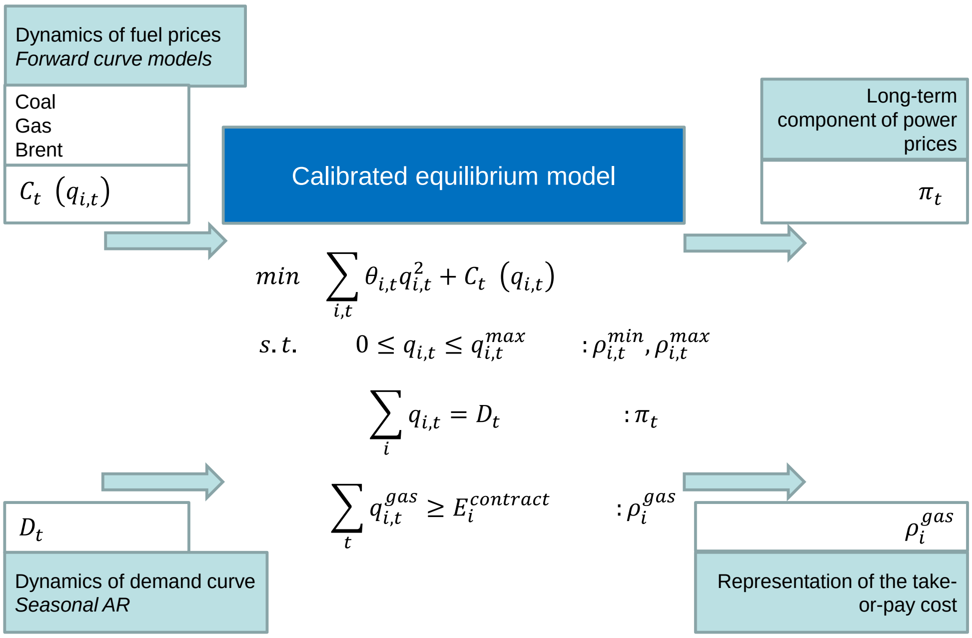

FIGURE 1. THE MODEL SCHEME.

As for the fuel price description, we will consider separate models for the evolution of coal, heating-oil and gas prices. The rest of prices –namely the corresponding to nuclear plants– are modeled as a known variable cost. The model for the evolution fuel prices is the model of forward curves proposed in (Clewlow & Strickland, 1999).

In addition, we model the power demand directly, instead of as a function of the temperature or humidity. The main reason for that choice is that there is often little trading activity concerning the primary drivers, and then there is no market information available. Thus, the model for power demand is based on an autoregressive process, combined with a deterministic seasonal component. We use the Linear Hinges Model, developed in (Sánchez-Úbeda & Wehenkel, 1998) and (Sánchez-Úbeda, 1999) as the estimator of the seasonal component.

In addition, hydro and wind production are modeled as known production in the system. Therefore, the demand values faced by equilibrium model should be thought of as the thermal demand of the system. That is, the system demand discounting the hydro production and the wind generation.

The model for the short-term component, y_t will be a discrete-time autoregressive process with weekly seasonality. This model allows capturing the equilibrium-reverting behavior of power prices.

3. Application to a real-size system

In this section, we will show the model performance in a case study of a real-size system. This case study will be a model of the Spanish market in the first eight months of 2008[3].

The Spanish system will be modeled using eighty-five power plants. They are classified under four different categories: coal, gas, fuel units and “other” units (the last term refers mainly to nuclear plants). Figure 2 represents the thermal plants considered.

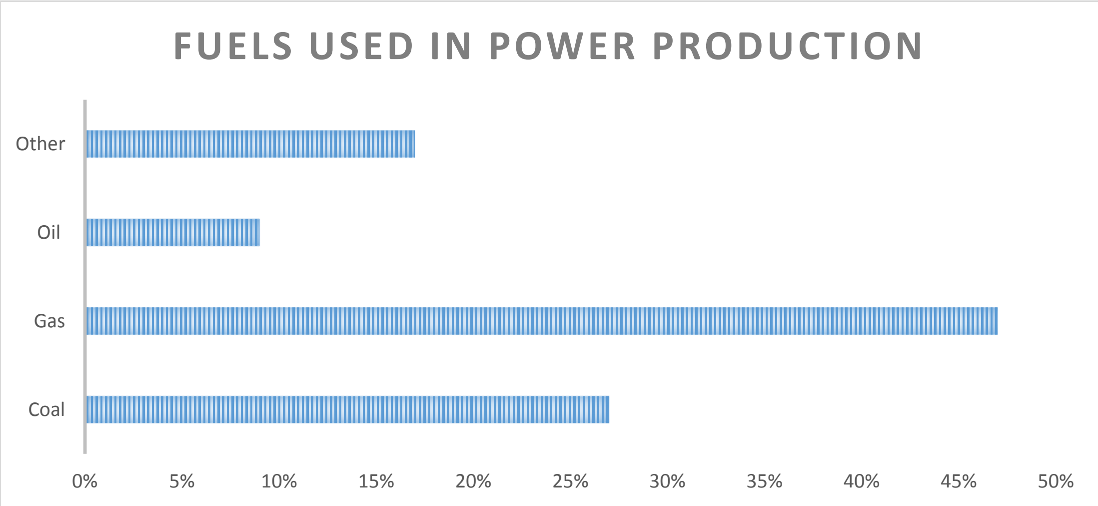

FIGURE 2. THERMAL PLANTS WITH RESPECT TO THE FUEL USED TO PRODUCE ELECTRICITY.

In addition, it is necessary to transform the fuel prices into variable costs. We model such transformation, in the study, as the price of just one contract of the complete forward curve, multiplied by the efficiency of the plant. In particular, the variable cost will be the forward price of the contract expiring in three months, multiplied by the efficiency. The rationale behind this is that power producers need at least three months to get additional fuel, and their variable cost is the cost of refueling. Furthermore, seven firms will be considered: Endesa (EN), Iberdrola (IB), Unión Fenosa (UF), Hidrocantábrico (HC), Viesgo (VI) and Gas Natural (GN). They own the thermal portfolio represented in Figure 3.

[3] This choice is a convenient one, as this is the time scope used in the case study of the basic model developed in (Vazquez & Barquin, 2013).

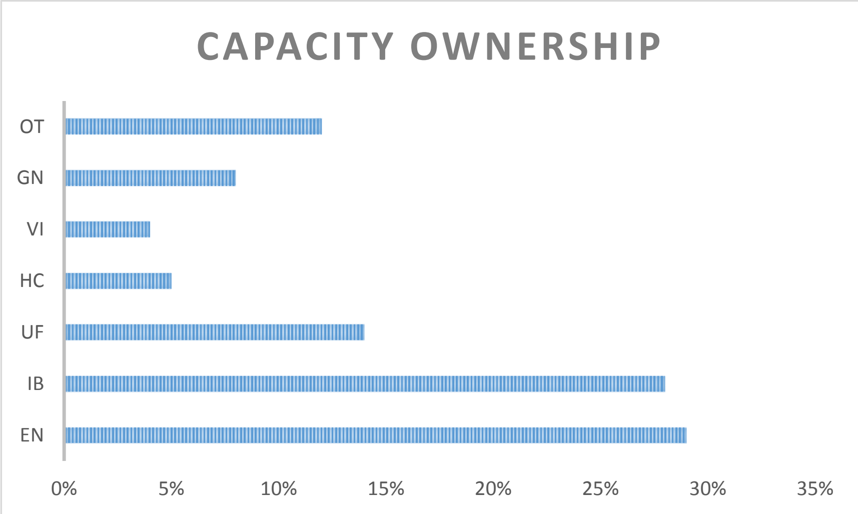

FIGURE 3. THERMAL PORTFOLIO OWNED BY THE FIRMS IN THE SYSTEM.

In addition, the case study assumes that no firm enters into gas contracts but Endesa[4]. This assumption is made in order to facilitate the analysis. From this point of view, the case study analyzes Endesa's decision-making process with respect to its gas contracts.

The simulation consists of 100 scenarios of the underlying factors, and 100 scenarios for the short-term factor. Both sets of scenarios are combined so that every short-term perturbation is added to each long-term scenario, resulting in a set of 10000 price scenarios. The details of the simulation procedure, including the calibration of the model, can be found in (Vazquez & Barquin, 2013).

2.1 Numerical results

Figure 4 shows the cost of the take-or-pay clause ρ_i^{gas} for several volumes of gas contracted. The probability considered in the figure is the probability that the take-or-pay cost is equal to or greater than each value. That is, Figure 4 represents the shadow price of the gas constraint (with the opposite sign), sorted in increasing order. In Figure 4, it can be observed the effect that, when the production obligation implied by the take-or-pay increases, the flexibility in the operation of power plants decreases, so that the cost associated with the contract increases.

To analyze contracting decisions, let us start by considering that available gas contracts are based on spot prices. That is, a typical situation involves take-or-pay contracts that specify a variable price, which is indexed to some extent to a certain fuel price index. In this regard, the extreme case of such indexation is a contract that specifies a price made up of the spot price –e. g. the Henry Hub price– plus some risk premium. In this case,

[4] Note that we refer to Endesa just in the context of the system modeled as a part of the case study.

the power producer negotiating the contract would face the problem of deciding the required risk premium.

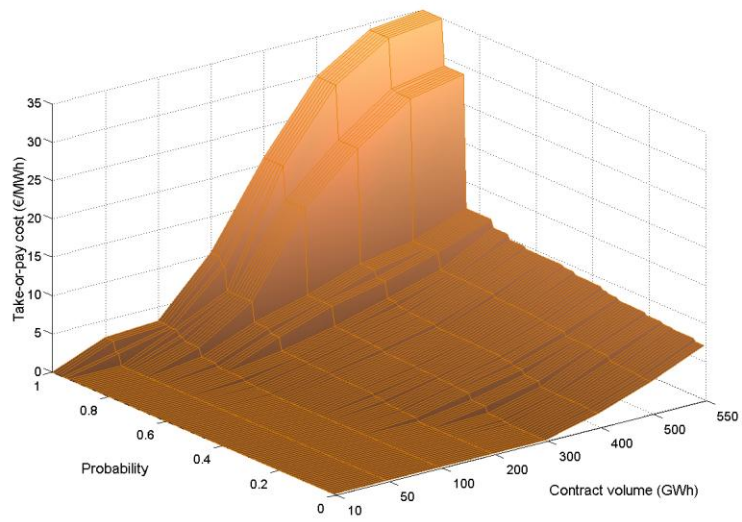

FIGURE 4. COST OF THE CONSTRAINT ASSOCIATED WITH THE TAKE-OR-PAY CONTRACT.

This risk premium can be identified with a reduction on the contract price equal to the gas constraint cost. Figure 5 represents the contract cost probability for different volumes associated with the take-or-pay.

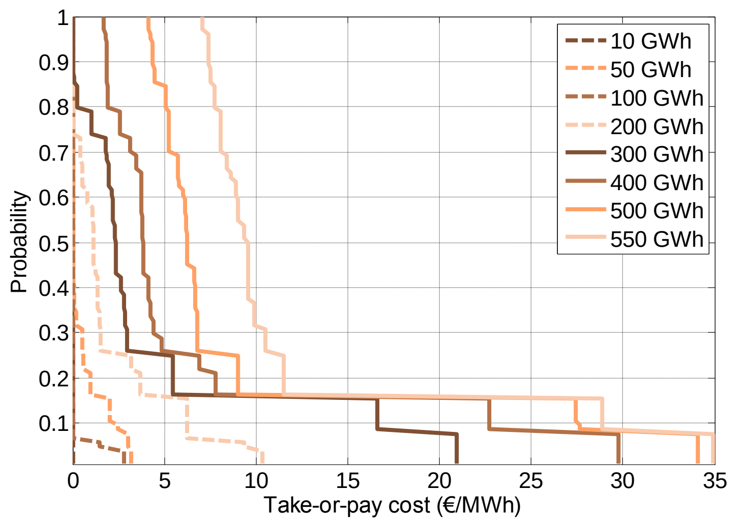

FIGURE 5. TAKE-OR-PAY COST FOR SEVERAL CONTRACT VOLUMES.

Each of the curves in Figure 5 shows the probability that the cost of the contract is equal or greater than the values in the horizontal axis. That is, the take-or-pay cost will be greater than zero with probability one –for the volumes used in the figure, and greater than 35 €/MWh with probability zero. A typical decision is related to the choice of the volume associated with the contract. For instance, consider that the firm is interested in entering into a contract with a risk premium that compensates for the take-or-pay cost 80% of scenarios. In this situation, it is possible to analyze the relation between premia and volumes considering the points of Figure 5 with probability 20% –i. e. the actual contract cost will be only greater than this value 20% of scenarios.

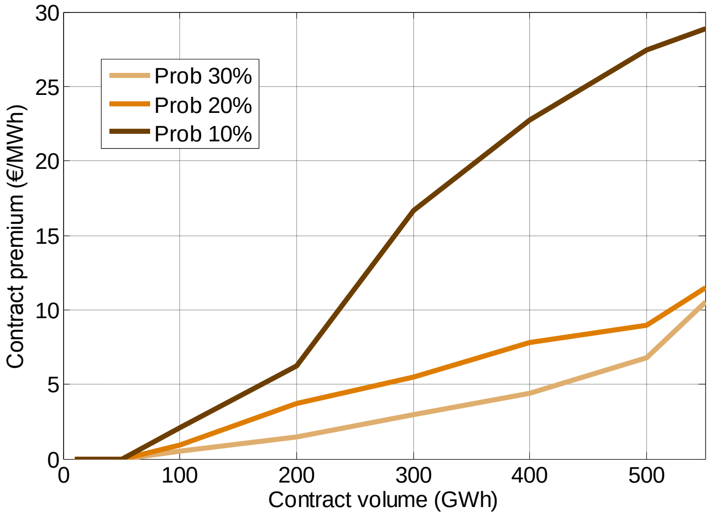

FIGURE 6. CONTRACT COST ENSURING TO COMPENSATE FOR CONTRACT COSTS.

Figure 6 shows the previous relation for different values of probability. It can be noted that the lower the probability (and hence the lower the exposition to adverse take-or-pay costs), the greater the contract risk premium. Analogously, large contract volumes imply large risk premia.

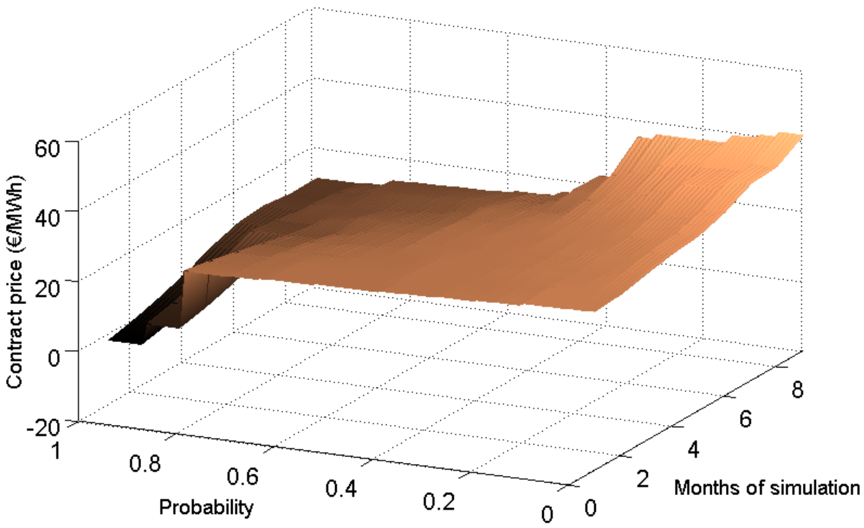

FIGURE 7. REQUIRED CONTRACT PRICE FOR THE CASE OF 550 GWH.

On the other hand, take-or-pay contracts often specify a fixed price for the whole contract period. That is, instead of indexing their prices to some spot gas index, contracts imply gas delivery at some fixed price at each point in time. Ideally, producers are willing to define contract prices as the ones represented in Figure 7, which shows the required contract price to compensate for the additional gas constraint.

As we are assuming in this case that the spot price is defined in a monthly basis, the opportunity cost associated with the contract varies from one simulation month to another. In this regard, Figure 7 represents the take-or-pay price defined by $p_t^{contract} = p_t^{gas} + \rho_i^{gas}$.

However, power producers are not often able to write a contract with variable price, but they have to specify a fixed price for the entire horizon. In this situation, firms face another source of uncertainty, which is related to the volatility of gas spot prices.

If $p_t^{contract}$ is the optimal price, and $p^{contract}$ is the fixed price of available take-or-pay contracts, it is possible that $p_t^{contract} > p^{contract}$ for some months of the simulation scope, whereas $p_t^{contract} < p^{contract}$ for others. Consequently, power producers face a trade-off between the risk of too high contract prices for some months (which implies an increased cost) and too low prices (which makes the negotiation of the contract harder). Figure 8 shows the kind of problem faced by power producers during the negotiation process.

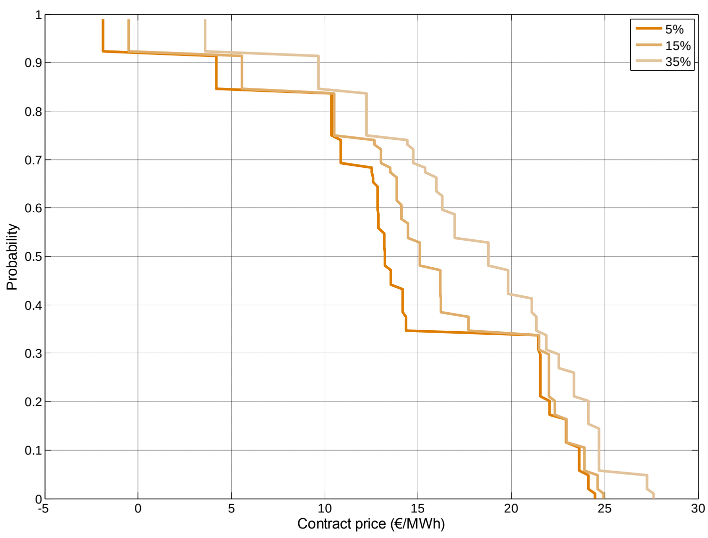

FIGURE 8. SEVERAL CURVES FOR DIFFERENT MEASURES OF RISK, FOR A VOLUME OF 550 GWH.

In the previous figure, we have assumed that agents' preferences can be represented by the probability that$p_t^{contract} < p^{contract}$. Specifically, Figure 8 represents that, for 5%, 15% and 35% of simulation periods, the price of the contract is lower or equal to

$p_t^{contract}$. In addition, the volume of the take-or-pay contract is 550 GWh. Hence, Figure 8 shows the probability that the required contract price is equal to or lower than the values in the horizontal axis. In contrast to Figure 5, this figure shows the required price of the take-or-pay contract, instead of just the associated risk premium. In this case, in addition, these values are true for the specified percentage of the simulation periods. In addition, assuming a probability of 5%, the curves for several contract quantities are represented in Figure 9. Finally, assuming the 5% case for the time risk, the resulting curves describing the price for several contract volumes and several risk decisions for the price risk are represented in Figure 10.

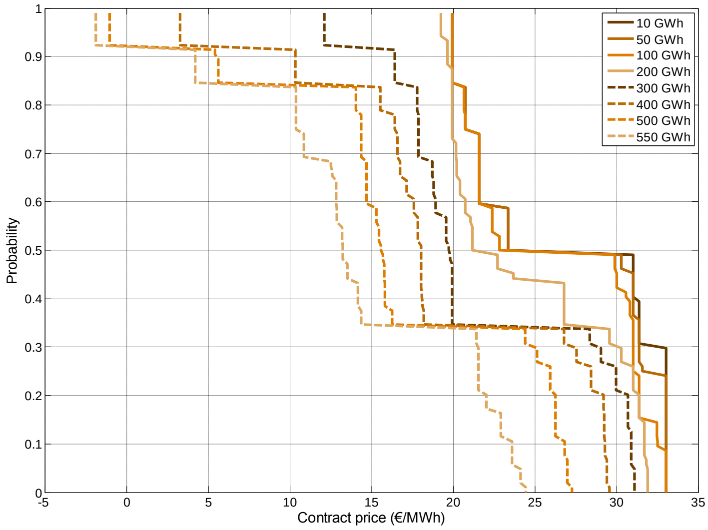

FIGURE 9. PROBABILITY CURVE FOR EACH QUANTITY UNDER TAKE-OR-PAY PROVISIONS.

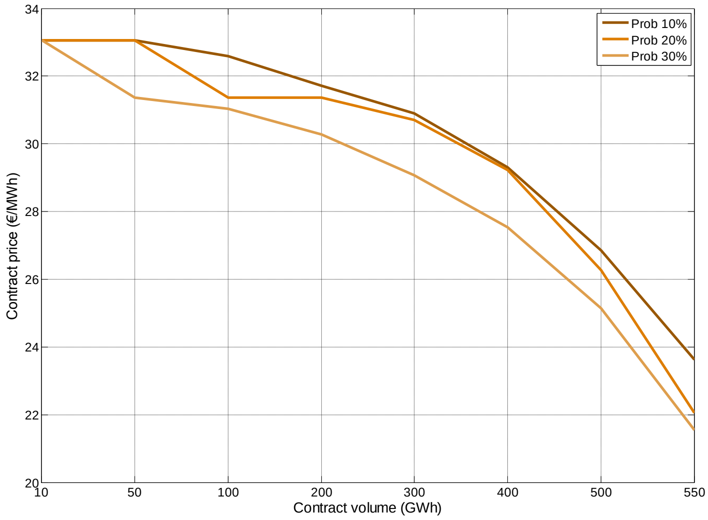

FIGURE 10. CONTRACT PRICE FOR SEVERAL RISK POSITIONS.

Therefore, when the take-or-pay price is forced to be unique for the entire horizon, agents must choose, besides to the probability that the risk premium compensates for the increased production cost, the probability regarding the time dimension.

4. Conclusion

Short-term flexibility is one of the central elements in the coordination of gas and electricity industries. However, the timing for gas purchases does not match with the timing for power production, so the ability to renegotiate gas purchases is central for power producers.

We have shown that the lack of short-term liquidity in gas markets creates a cost for power producers purchasing gas in the long run. When power producers cannot undo their gas positions in the short run, they see take-or-pay provisions as sunk costs.

Therefore, the production with gas-fired power plants becomes inefficiently priced, and thus the merit order in power markets is affected. Consequently, power prices are paying for the need for adjustment. Put it differently, when gas markets are illiquid, the cost of the inefficiency impacts directly on the power industry.

This gives insight to the trading arrangements observed in the natural gas industry. On the one hand, power producers look for adaptation, so they prefer the shorter-term contracts. Gas producers need to lock in gas production, so they prefer longer-term contracts. In this bargaining process, not only is the specificity of gas production relevant. If the duration of the contract is shorter, gas producers can sell gas at a higher price (as it is more valuable for power producers). On the other hand, power producers bear the price risk of the contract. The longer the duration of the contract, the larger the incentive of power producers to ask for low volatility.

Furthermore, other potential consequence of gas-fired power plant's risk assessment of long-term contracts has to do with system reliance. This aspect is becoming increasingly relevant both in the US and in South America. Under certain power generation portfolios, power plants are not able to bear the sunk cost associated with gas long-term contracts (both for 'gas commodity' and for transport contracts). This may drive power plants to short-term markets and hence it may give misleading signals regarding the gas infrastructure. Consequently, the electricity system reliability decreases. This phenomenon has been observed in New England in 2014, and solutions to it are currently under debate. In that view, developments of the model presented in this paper should investigate the incentives that intermittent generation from gas-fired power plants gives and how that impacts both power and gas systems.

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