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ESTIMATION OF BRAZILIAN QUARTERLY GDP WITH COINTEGRATION METHODS AND BENCHMARKING PROCESSES BY STATE SPACE MODEL

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ABSTRACT

This paper presents a methodology for estimating the Brazilian GDP quarterly series in the period between 1960-1996. Firstly, an Engle-Granger's static equation is estimated using GDP yearly data and GDP-related variables. The estimated coefficients from this regression are then used to obtain a first estimation of the quarterly GDP, with unavoidable measurement errors. The subsequent step is entirely based on benchmarking models estimated within a state space framework and consists in improving the preliminary GDP estimation in order to both eliminate as much as possible the measurement error and that the sum of the quarterly values matches the annual GDP.

Key Words: benchmarking, Engle-Granger's equation; Kalman's filter; state space models; GDP.

JEL: C32, C51, C52, E01.

1. Introduction

This paper describes and applies a methodology for estimating the quarterly Brazilian GDP from 1960 to 1996. The methodology is based on the initial estimation of the quarterly GDP, obtained through an Engle-Granger static equation using GDP annual data and GDP-related variables, namely: automobile and cement production, industrial consumption of electricity in the Rio de Janeiro–Sao Paulo axis, and the national treasury tax revenue. The estimated coefficients of this regression are then used to build a quarterly equation between GDP and those variables.

This equation produces a first quarterly GDP estimation, henceforth referred to as "dirty" GDP – due to unavoidable measurement errors. The next step consists in improving the estimation, ridding it as much as possible from measurement errors and making it consistent with the annual GDP calculated by the Brazilian Institute of Geography and Statistics (IBGE), an official agency, to clarify: *the sum of the quarterly estimations must be equal to the year total.*

The process of harmonizing quarterly and annual estimations is called benchmarking, constantly used by official data agencies. In this paper, the benchmarking procedure adopted is anchored on the state space modeling, in which a time series is broken into unobserved and directly interpretable components (cf. Harvey, 1989; and Durbin & Koopman, 2001).

The paper is organized as follows. Section 2 briefly discusses the benchmarking process and reviews the literature on the subject. Section 3 describes the quarterly GDP estimation methodology. Section 4 introduces and discusses the estimated models' results, and Section 5 is the conclusion of the article. The Appendix describes the construction of the series used and shows the results of the unit root tests employed.

2. Benchmarking

A common problem with official statistics is the adjustment of monthly and quarterly observations obtained through surveys or sampling and therefore subject to errors. The adjustment is made with annual data from censuses or more detailed surveys, hypothetically presumed to be free of sampling errors. The annual total is called the benchmark; the process of harmonizing estimations with the year total is called benchmarking.

More specifically, benchmarking – or harmonization – consists in conveniently matching two measurement sources of the same time series, usually obtained from distinct frequencies. The lowest-frequency series – the benchmark series – is assumed as having a more reliable registry. Benchmarking is the process of trying to adjust the highest-frequency series to the benchmark one. This is done by breaking down the series into its structural elements – trend, seasonality, cycle, irregularity and measurement error – and from the sum of those elements excluding the error.

There are two major methodologies to apply benchmarking to a time series: a purely numeric approach and a statistic modeling one. The numeric approach differs from the statistic modeling by not specifying a statistical model to the studied series. The numeric approach encompasses the family of methods based on the minimization of a squared sum proposed by Deaton (1971) – following the principle of movement preservation –, Bassi (1958), and Ginsburgh (1973). An application of such procedure can be found in Di Fonzo & Mariani (2003). The statistic modeling method, in turn, involves models based on ARIMA processes proposed by Hillmer & Trabelsi (1987), state space models proposed by Durbin & Quenneville (1997), and models that use a group of regressions such as Cholette & Dagum (1994), Mian & Laniel (1993) as well as the references cited by these authors.

3. Paper methodology

3.1 First step: obtaining "dirty" GDP

In the lack of official statistics collected for computing GDP, the assembly of quarterly estimates for the actual accrued product can adopt three criteria: (i) annual data interpolation; (ii) own survey from samples of goods and services; and (iii) a combination of the first two criteria; cf.

Contador & Santos Filho (1987). In Brazil, researchers preferably adopt interpolation with GDP-related series. In this paper we shall follow such procedure by using the series used by Cardoso (1981), namely: automobile and cement production, industrial consumption of electricity in the Rio de Janeiro–Sao Paulo axis, and the national treasury tax revenue.

Computation of the "dirty" quarterly GDP is based on the estimation of a GDP regression against the mentioned series, with annual-frequency series being expressed in 1980-based indexes with the purpose of obtaining long-term relationship coefficients between the variables. The estimation method used was Engle-Granger's two-stage cointegration procedure. Having estimated a cointegrating vector by OLS we form a linear combination by using the previously listed quarterly frequency series in order to obtain the interpolated series of the quarterly GDP. The estimated quarterly series is chained with IBGE's series in 1980, producing the "dirty" GDP series, which shall be perfected in the second half of the procedure.

Prior to 1980, GDP data are available only in annual frequency, which explains why the estimation was performed in this frequency. At first, the estimation aims to recover GDP data for the 1964-1979 period. However, given the few observations, the period was increased to 1960-1996. These dates are associated to the series availability and to the change in IBGE's quarterly GDP investigation methodology.

Although at first sight the variables selected for initial estimation might seem questionable, further analysis shows that they represent the chief economically productive sectors in ad adequate way. Since 1960, the automobile industry has been Brazil's most important economic segment. Cement consumption, in turn, is an suitable representation of the rank of civil construction activity, an area that generates a large portion of employment in the economy. Likewise, industrial electricity consumption in the country's major economic region (the Rio de Janeiro–Sao Paulo axis) is an added relevant index of economic activity. Lastly, it seems obvious that the federal tax revenue should suitably represent governmental activities, especially when it is considered that a part of it is passed on to states and cities, being entirely spent in a wide portion of the studied period.

Thus, “dirty” GDP is the first approximation of the quarterly GDP that recovers missing data prior to 1980, and that has thenceforth a seasonal pattern identical to that of the IBGE’s series.

3.2 Second step: state space models for benchmarking and obtaining the “clean” GDP.

The general linear Gaussian state space model (SS) is defined by the following equations:

$$y_t = Z_t\alpha_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, H_t),$$

$$\alpha_{t+1} = T_t\alpha_t + R_t\eta_t, \qquad \eta_t \sim N(0, Q_t) \qquad (3.1)$$

$$\alpha_1 \sim N(a_1, P_1).$$

y_t is a of *p* x 1 vector of observations; α_t is called the state vector, is unobservable and its dimensions are *m* x 1; ε_t and η_t are independent error terms; matrices of system Z_t, T_t, R_t, H_t and Q_t, general, contain unknown parameters, which are assembled in a ψ parameters vector. The model estimation in (3.1) is done with Kalman filter (KF) (state vector) combined with maximum likelihood (parameters vector). KF is formed by a set of equations that estimate, recursively in time, the average and conditional variances of the state vector. For further details about those equations, their deductions and their combination with maximum likelihood estimation, see Harvey (1989, chapter 3) and Durbin & Koopman (2001), chapters 4 and 7.

State space models for benchmarking problems have been studied quite broadly by Durbin & Quenneville (1997). One of the corresponding forms in SS proposed by Durbin & Quenneville was later reviewed by Durbin & Koopman (2001), chapter 3. In this paper, a probabilistically equivalent version of Durbin & Quenneville’s model is presented. Taking y_t as the “dirty” quarterly GDP (theoretically with measurement errors) and x_t as IBGE’s annual GDP (theoretically free of measurement errors), observations must be y_t if time t is not a multiple of four, or $(y_t, x_t)'$ in case time t is a multiple of four (due to the quarterly frequency, this indicates the turning of a new year). The state vector, entirely inspired in the structural model approach for time series (cf. Harvey, 1989) is given by the expression $\alpha_t \equiv [\mu_t \quad \mu_{t-1} \quad \mu_{t-2} \quad \mu_{t-3} \quad \gamma_t \quad \gamma_{t-1} \quad \gamma_{t-2} \quad \gamma_{t-3} \quad \varepsilon_t \quad \varepsilon_{t-1} \quad \varepsilon_{t-2} \quad \varepsilon_{t-3} \quad \xi_t]'$, where μ_t is a local level, γ_t stands for stochastic seasonality, ε_t is an irregular component and ξ_t is a measurement error associated to the "dirty" GDP, which admittedly follows an AR(1) stationary process. Matrices Z_t must be $Z_t = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1]$ if t is not a multiple of four, and $Z_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$ if t is a multiple of four. Matrices H_t are null for all moments in time. The other matrices associated to the state vector equation and time-invariant for this model must follow the postulated evolutions for the state vector components; their full expressions are presented in Durbin & Quenneville (1997). Notice that, given the SS formulation, the consistency between "clean" GDP (defined by $y_t^* = y_t - \xi_t$) and IBGE's annual GDP always occurs in multiples of four periods.

The structural components obtained from this procedure and used in this paper are the following: trend, seasonality, irregularity and measurement error. "Clean" GDP is the sum of these components below the measurement error.

Methodology – a summary

The steps described in the two previous subsections define the following algorithm:

1) Estimate a relationship between the annual GDP and variables either related to it or that explain its behavior. In case the variables are integrated, the simplest estimation method is Engle-Granger's two-stage cointegration test. All series must also be available at a quarterly frequency.
2) Test the non-cointegration hypothesis by ensuring the existence of a unit root in the cointegration regression residuals. In case such hypothesis is accepted, return to the

previous item and specify other independent variables. In case co-integration prevails, advance.

3) Use the estimated equation coefficients with annual data to interpolate quarterly GDP before 1980, multiplying them by the respective series of the independent variables in quarterly frequency.
4) Link the interpolated series in the above item with the quarterly GDP computed by IBGE as from 1980. Name the quarterly series resulting from such link the "dirty" GDP.
5) If the "dirty" GDP is estimated in indexes, convert it into values, to avoid non-linearity in the next step, by multiplying the index series by the average quarterly real-GDP of a representative year in the sample, *e.g.*, 1980.
6) With the "dirty" GDP series in real values, estimate the SS model described in (3.1). Here the structural components are obtained: trend, seasonality, irregularity and measurement error. The generated series presents the measurement error to be extracted.
7) Check the model's basic assumptions through residuals analysis, paying particular attention to the presence of unconditional heteroskedasticity, a common violation in econometric studies involving macroeconomic time series. If heteroskedasticity is confirmed, normalize the "dirty" GDP by dividing it by a macroeconomic series that may, at least, be co-responsible for the variance inconstancy, and re-implement step 6 with the normalized "dirty" GDP. If heteroskedasticity is not confirmed, advance.
8) Obtain the $y_t^* = y_t - \xi_t$ series by adding the structural components extracted below the measurement error.
9) In case there has been standardization in step 6, multiply y_t^* by the normalizing macroeconomic variable in step 7 and name the resulting series the "clean" GDP. If there has been no standardization, consider y_t^* as the "clean" GDP.

One final comment about algorithm steps 6 and 7: preliminary experiments actually indicated the presence of strong heteroskedasticity related to the economic growth rate as well as the inflation rate in the studied period (1960 to 1996). The series chosen for "dirty" GDP normalization should be such that it keeps a relation between GDP and inflation. The alternative was to use the average annual index[1] of the real tax revenue. In sum, the SS model (3.1) was estimated with the "dirty" GDP (in values) divided by the real tax revenue average yearly index. The normalized "dirty" GDP series in values is displayed in Figure A.1 in the Appendix. The described procedure is summarized in the flow chart below.

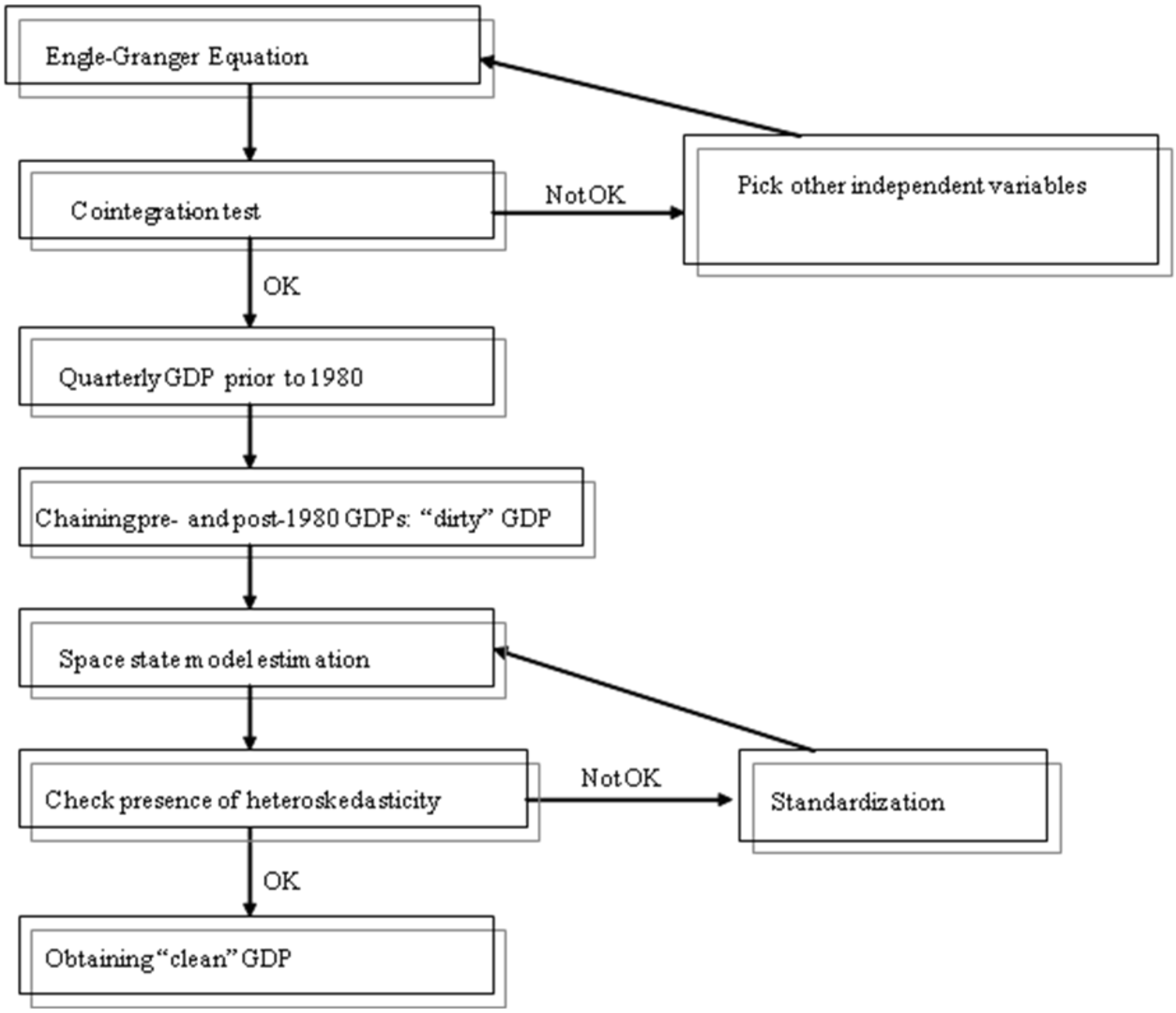


Figure 1 - Proposed methodology flow chart.

[1] The reason for using the index variable is that the real revenue expressed in 1980 currency has very low values due to the monetary reforms that took place in the sample.

4. Results

4.1 "Dirty" GDP

Engle-Granger's regression estimation results, associated to step 1 in the methodology are reported in Table 1. IBGE's GDP index is the dependent variable. On the right side of the equation there is a constant term (C), a linear trend (TT), and the electricity consumption (IEES), automobile production (IAUTO), real tax revenue (IRTNRS) and cement production (ICIM) indexes. R^2 and DW statistics are also informed. Then the statistics for the presence of unit root in the static equation residuals and the number of lags used in the unit root test appear together with portmanteau tests' p-values for the presence of a serial co-relation and value of the Bayesian information criterion for step 2 in the procedure. At 10% the null hypothesis for non-cointegration between variables is rejected. The test in step 2 shows residuals without serial co-relation according to the reported Ljung-Box (Q) tests.

Table 1
Results of Engle-Granger procedure between the annual GDP and related variables

Variables	Estimated coefficients
C	0.112423
TT	0.007836
IEES	0.283306
IAUTO	0.108377
IRTNRS	0.098286
ICIM	0.231170

R^2=0.9984	DW=1.5793	$t_{\hat{\alpha}}$ =-4.6341†	lags=0	Q(1)=0.931
Q(4)=0.228	Q(8)=0.479	Q(12)=0.126	SIC=-5.4545	

Note: (†) stands for rejection of the null hypothesis of a unit root to the significance level of 10%.

Although the amount of observations is not enough to support arguments towards superconsistency of the OLS estimates, the GDP series estimated through this equation – and chained since 1980 with IBGE's series – will be adopted as initial proxy for the quarterly GDP index – the "dirty" GDP. Figure 2 shows the "dirty" GDP series, which is still inconsistent with the annual total calculated by IBGE.

FIGURE 2: "DIRTY" GDP INDEX - QUARTERLY DATA

4.2 "Clean" GDP

Table 2 and Figures 3 and 4 below show the statistics referring to the estimated benchmarking state space model for the "dirty" GDP series. All tests have been performed with the model's standardized residual series (so-called *innovations*; cf. Durbin & Koopman, 2001, chapter 4), and together with the panels in Figure 3 point that the model's basic assumptions are being followed, except for the normality. This is not an actual problem, since the statistic inference over unknown parameters is not within the scope of this paper. Heteroskedasticity was entirely treated, which reveals that the tax revenue satisfactorily captured the variance inconstancy. It is also pointed out that no signs of breaks were found in the level term of the resulting series – the “clean” GDP. Neither was the presence of outliers noticed around 1980 – a particularly critical year, in which the chaining of the estimated series and the one published by IBGE[2] took place. Lastly, it must be observed that the extracted seasonal component (Figure 4) does not show any significant changes around 1980 (observation 105). What can be observed is an oscillation of the seasonality throughout the period. This allows one to deduce that the chaining **per se** of the estimated series in step 1 with that of IBGE did not cause any material changes in the seasonal pattern.

TABLE 2
Analytical information about the SS model estimation with benchmarking. Values in brackets are p-values of the corresponding tests.

Log-Verisimilitude	229.262
F Test for heteroskedasticity	0.7382 [0.8747]
Durbin-Watson	1.8561
Ljung-Box Test (12 *lags*)	12.964 [0.3717]
Jarque-Bera Test	21.895 [0.0000]

2 To confirm such statement, a “clean” GDP series regression was estimated in levels and differences against deterministic terms (constancy, trend and seasonal dummies) by recursive least squares. Coefficients, as well as their confidence intervals did not show any change in 1980. That stability analysis is not presented in this paper.

FIGURE 3: STARDARDIZED INNOVATIONS STATISTICS

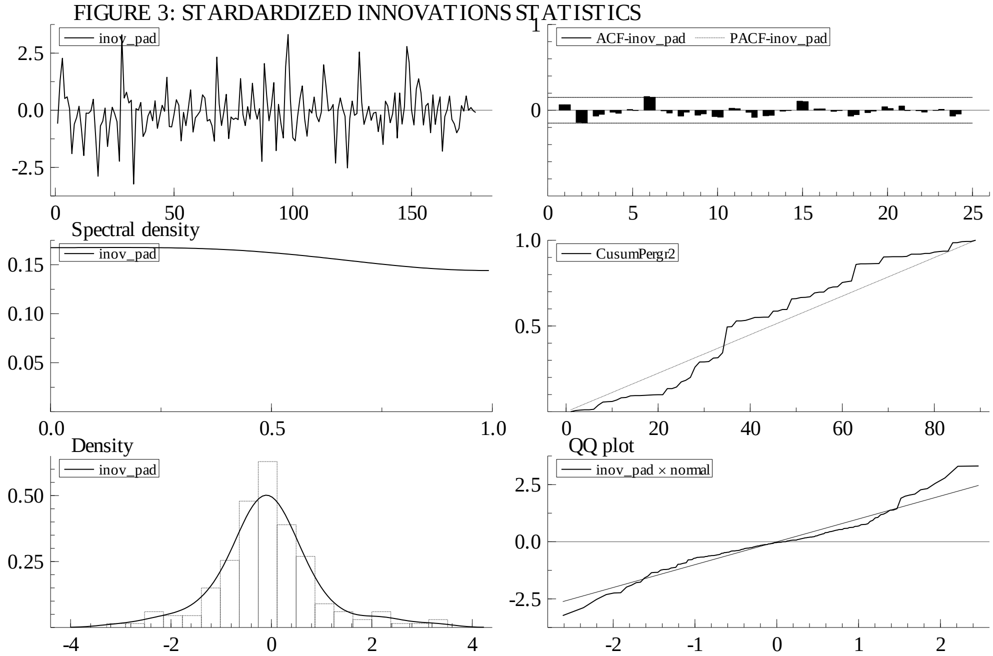
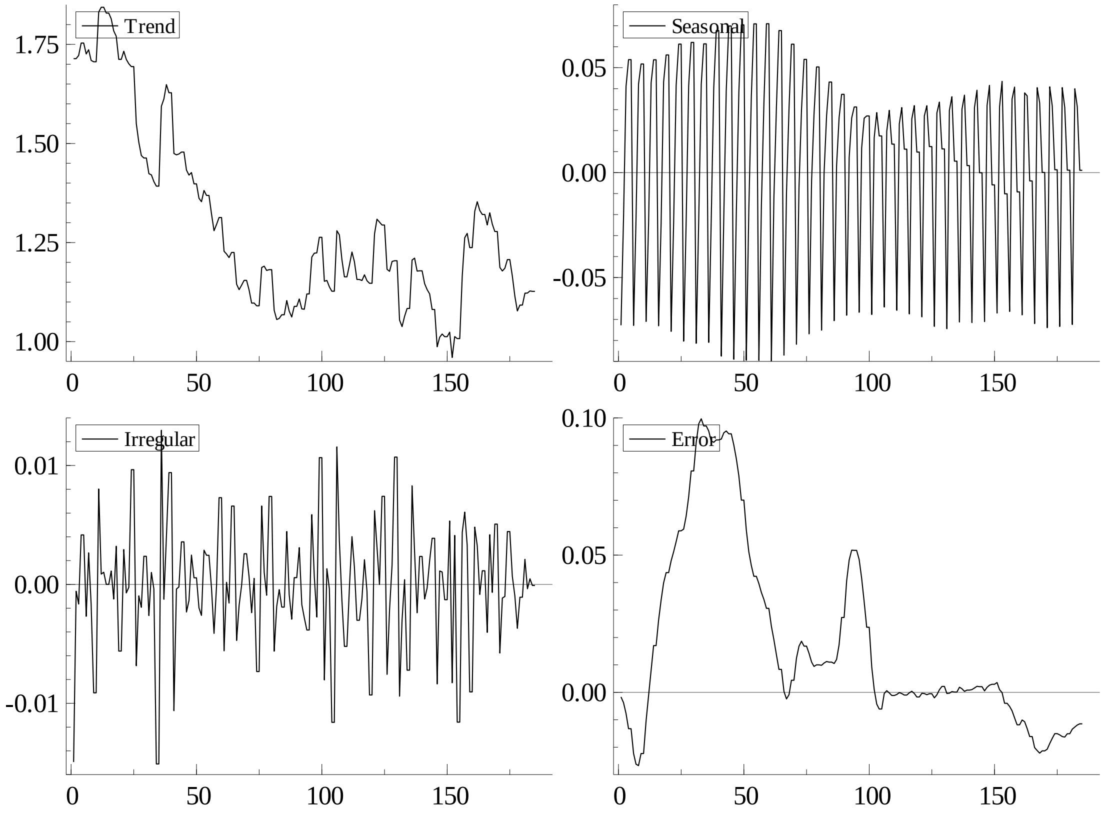


FIGURE 4: ESTIMATED GDP COMPONENTS NORMALIZED BY TAX REVENUE



The one-step-ahead forecast evaluations made for the 1980-1996 period are displayed in Table 3. It is the period to which IBGE's official computations are available. According to the selected statistics the one-step-ahead forecasts are accurate and a strong evidence of the model's adequacy to the data.

TABLE 3
Forecast evaluation for the 1980-1996 period

MSE Squared Root	0.0112
MSE Percentage	0.0078
Mean Absolute Error	0.0073
Mean Absolute Percentage Error	0.0052
Pseudo-R2	0.9979

4.3 Discussion

Among the limitations of the results it is pointed out that the estimated measurement error's first-order serial correlation coefficient is very close to one[3]. This strong persistence – see fourth panel in Figure 3 – is interpreted as "inherited" from structures that should potentially be embedded in the set of the other state vector components.

Two particular results are highlighted. The first – not reported in this version of the paper – is that the estimated model without heteroskedasticity treatment yields virtually the same "clean" GDP series, with no material differences found between them. Thus, for this SS model for benchmarking applied to the "dirty" GDP series, Kalman's filter has proved to be robust to this type of violation of the basic assumptions.

[3] Unit root tests no longer reject, as a set, the null hypothesis that the series has integration order 1 (one).

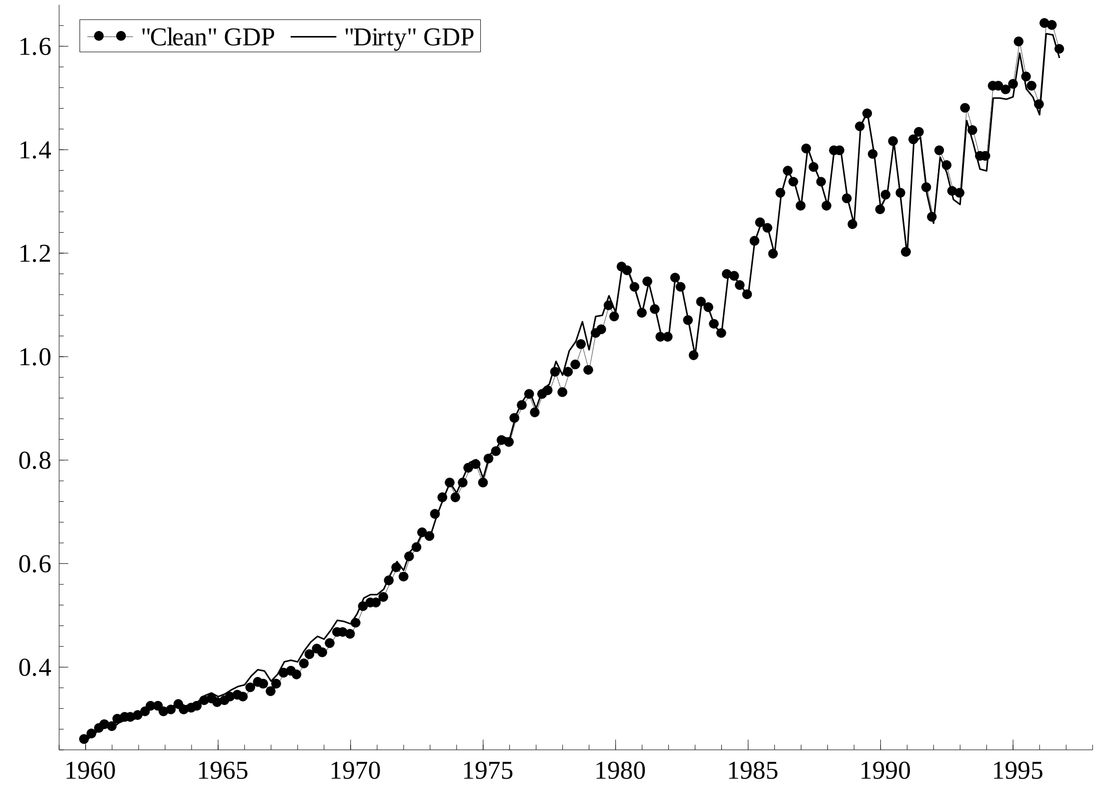
The second – see Figure 5 – is the systematic overestimation as from 1990.4 of the "clean" GDP in face of the "dirty" GDP. Here the bias is characteristic of the estimated official quarterly GDP. Therefore, it empirically demonstrates the inconsistency between quarterly and annual amounts for IBGE's estimations. This allows the conclusion that the proposed procedure is correcting the official quarterly estimations by harmonizing them with the annual GDP.

5. Conclusion

This paper focused on: (i) proposing a methodology for estimating the quarterly GDP for the 1960-1996 period; (ii) empirically demonstrate that methodology; and (iii) report and analyze the resulting series. The importance of a judicious proxy for the quarterly GDP for periods before 1980 arises from the need to carry out empirical studies that could cover a longer period of the Brazilian economic history. The results have proved quite satisfactory regarding the diagnosis of the residuals and the predictive power of the model. The "clean" GDP estimated with benchmarking corrects, to the maximum, the measurement errors found in the "dirty" GDP. Consequently they are consistent with the annual GDP and provide, for that reason, estimates with an additional attractive factor when compared to that presented by IBGE, the official Brazilian source.

For further studies in this niche, two directions are presented. The first one is to apply the methodology for benchmarking multivariate models, which would theoretically bring higher precision and reliability to the final quarterly GDP series. The second one regards to use the methodology to obtain quarterly GDP forecasts.

FIGURE 5: "DIRTY" AND "CLEAN' GDP



Appendix: Data used and unit root tests

The series used in the paper were subject to consistency analysis, and when they appeared in more than one source, they were compared in such a way as to identify and correct typing and/or calculation errors and make precision as high as possible. Basically, this comparison was made between the series published at the following websites (see bibliography): The Brazilian Central Bank (Banco Central), IBGE, IPEADATA, Conjuntura Econômica, as well as the author's own databank built throughout the years. As a rule, the data was considered to be used are those most recently disclosed in an official publication. In this case, except for some extraordinary review, data could be considered as definitive. Figure A1 contains data in quarterly indexes (base 1980) of automobile production (IAUTO), cement production (ICIM), industrial consumption of electricity (IEES), and real tax revenue (IRTNRS).

FIGURE A1: GDP-RELATED VARIABLES

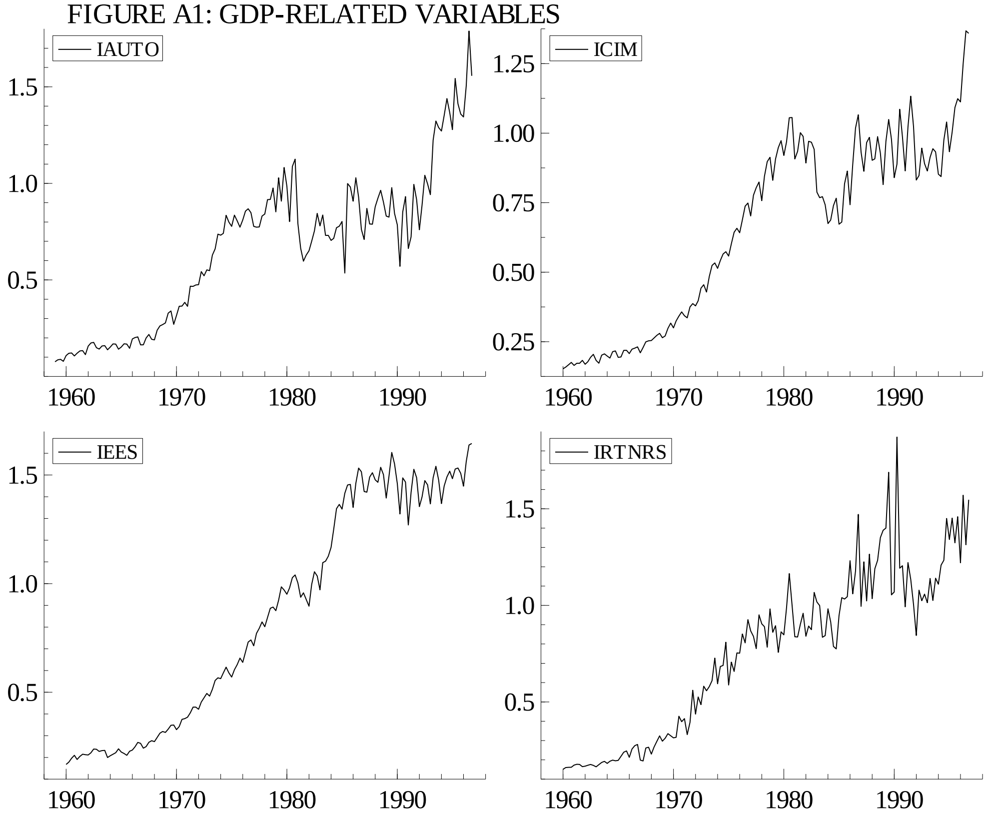


Figure A2 shows the “dirty” GDP series divided by the real tax revenue series, adjusted and expressed in indexes (see text).

Table A1 reports the unit root test results for the annual series used in Engle-Granger’s regression for the 1960-1996 period. The first two are modifications to the ADF test, and the last shows four statistics that are modifications to the Phillips-Perron, Bhargava and ERS-PO statistics; see Maddala & Kim (2002). Considering the test results, it can be admitted that all series have integration order 1.

FIGURE A2: ADJUSTED "DIRTY GDP" - QUARTERLY DATA

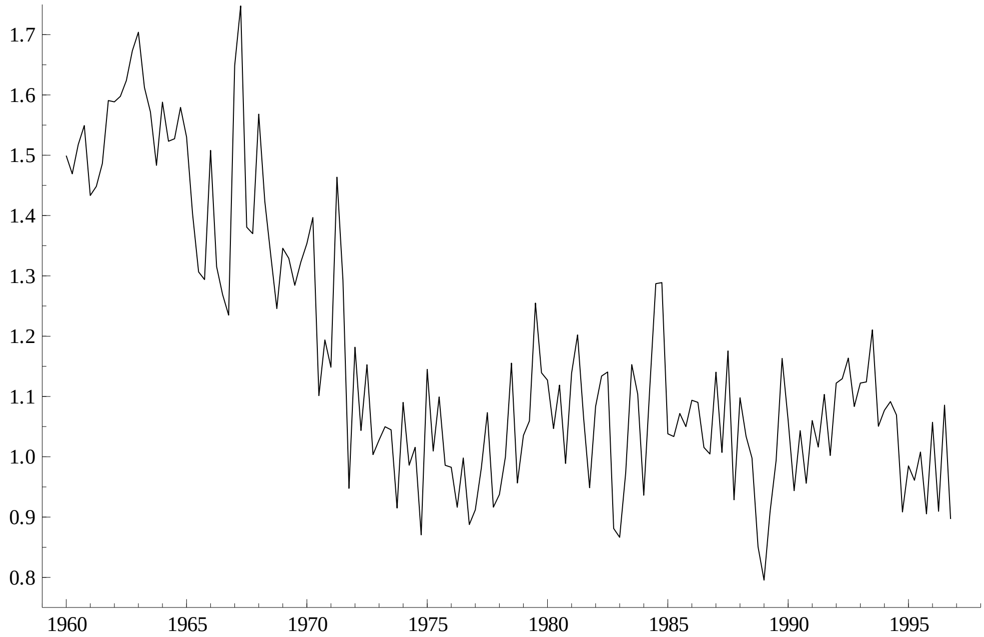


Table A1
Unit root test results

Test / Series	DF-GLS	ERS-PO	Ng-Perron			
			MZ_{α}^{d}	MZ_{T}^{d}	MSB^{d}	MP_{T}^{d}
GDP	$t\hat{\alpha}$ =-2.987	P_T=1.330	-18.128	-3.008	0.166	-1.360
	Lags=3	Lags=3	Lags=3	Lags=3	Lags=3	Lags=3
AUTO	$t\hat{\alpha}$ =-5.350	P_T=1.458	-18.419	-2.984	-0.162	-1.513
	Lags=0	Lags=2.79	Lags=2.79	Lags=2.79	Lags=2.79	Lags=2.79
CIM	$t\hat{\alpha}$ =-2.849	P_T=2.738*	-15.950	-2.481*	-0.156	-2.739*
	Lags=3	Lags=3	Lags=3	Lags=3	Lags=3	Lags=3
EE	$t\hat{\alpha}$ =3.142	P_T=1.887*	-13.363*	-2.570	-0.192*	-1.891*
	Lags=1	Lags=1	Lags=1	Lags=1	Lags=1	Lags=1
RTNR	$t\hat{\alpha}$ =-5.507	P_T=1.419	=-16.625	-2.883	-0.173*	-1.474
	Lags=0	Lags=2	Lags=2	Lags=2	Lags=2	Lags=2

Note: (†) stands for rejection of the null hypothesis at significance level of 10%; (*) at 5%; the absence of a symbol means rejection at 1%. In all cases, the null hypothesis was rejected.

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